Casino Articles: Player Loss

PLAYER LOSS (How To Deal With Actual Loss In The Casino Industry)

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Most complimentary and junket programs utilise either theoretical win or turnover upon which to calculate player complimentaries or rebates. In some cases, particularly in the United States, junket programs rebate a percentage of loss to the junket organiser or player.

Dealing with actual loss, however, is a difficult and often misunderstood issue. A common premise is that "so and so is a born loser" or "the money's in the bank" or "they do it in Vegas (or at Caesars/Hilton etc)" and therefore giving a percentage of loss back to the player is alright.

The problem is that the percentage rebated is often plucked from the air and has no mathematical basis. This then leads to the fact that often no one knows what the theoretical or long term cost of such a policy is to the casino company.

In most business organisations understanding cost implications are a central premise to operating effectively. Casino Operations should be no different particularly in an area where the rebate is a totally hard cost.

Therefore, what is the cost of rebating a percentage of loss to a player or junket organiser?

Firstly, it is important to recognise that the theoretical loss by a player is a combination of all winning and losing events experienced by a player in a game for a given number of results. Because most Casino games are fundamentally biassed against the player, that result is a negative from the player's perspective. Simply, that result may be calculated by multiplying the house advantage by the number of decisions and the average bet. Rebating a percentage of theoretical loss takes into account therefore both winning and losing situations and provides a long term validity to the policy of rebating a percentage of theoretical loss.

Thus, when theoretical loss is dealt with factors such as average bet, time played, decisions per hour and house advantage are incorporated. However, when actual loss is being dealt with most policies only deal with the amount of the loss. It is critical that other factors such as the number of decisions are incorporated, as criteria are essential to ensure that the policy is valid.

This is because it is erroneous to believe that the percentage of actual loss rebated provides the same percentage of theoretical loss. In fact, if a policy rebates a fixed percentage of loss which is something greater than the house advantage, then the theoretical cost to the company will range from approximately the rebate percentage divided by twice the house advantage and would minimise at the rebate percentage. If the rebate on loss percentage were 10% and the house advantage 1.25%, then the theoretical cost of the rebate will range from roughly 400% of theoretical win at maximum, in an even chance game, and minimise at 10%.

The maximum cost would be realised if only one hand were played and then the player settled and were paid the rebate, with the minimum theoretical cost occurring if the player didn't settle until they had played a very large number of hands. Many would argue that no one would play only one hand and then settle or that of course no rebate would be paid under such a scenario. The real problem is that without play criteria it is the customer who may be in control of the net outcome. Much like the transition from paying complimentaries as a percentage of drop or credit line to basing these on calculations of theoretical Casino win, so to must rebate on loss policies change to mathematically sound business decisions.

When rebating on loss, what must be calculated is the conditional mean of all situations where the player loses. In all cases because we are dealing with biassed games that value will exceed or equal the mean of all possible events, both winning and losing, which we refer to as the player's theoretical loss.

If a rebate on loss policy is to be sound, it is a percentage of the latter which should be utilised to calculate an equivalent rebate on loss percentage for a given number of decisions. That can be accomplished by determining the percentage of theoretical loss relative to the conditional mean of only player losses.

In a simple one hand example on an even money game, if normally the Casino were prepared to pay back 50% of theoretical loss then for each $1 wagered the player would receive 50% of the house advantage multiplied by the number of decisions. If the edge were 1.2% then 0.6% of $1 would be paid back to the player regardless of whether they won or lost. If it were only the player who lost to be rewarded then that player could be provided nearly twice as much, as the net position would be compensated by the winning player receiving nothing. Why slightly less than double? Because the player would lose 50.6% of the time and thus paying 1.186% of actual loss if settlement occurred after a single hand would be the equivalent of paying 50% of theoretical loss for the example cited.

As the number of hands increases so to does the percentage of actual loss which may be rebated until such time as the number of hands played is so large that in virtually every instance the player loses and thus the rebate percentage on actual loss may equal the percentage of theoretical loss. This is due to the fact that in such a case the theoretical loss (mean) and the conditional mean are one and the same. If 50% of theoretical loss were the general policy to be returned, then the maximum rebate on actual loss would also be 50%.

How large a value for the number of hands would this take?

In an even chance game with a 1.2% house advantage the following could be calculated.

One standard deviation = square root (N)
where N is the number of hands

99.7% of all results fall within three standard deviations of the mean.

Therefore, 99.85% of all results would fall to the right of minus three standard deviations.

If we were to solve for when 0 were -3 standard deviations from the mean we find

3 square root (N) = mean

mean = N x edge

3 square root (N) = 1.2% N

3 = N
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1.2% square root (N)

Therefore N = (3/1.2%) 2
N = 62 500
Thus, if a player were to play approximately 62,500 hands and then settle it would be appropriate to pay 50% of whatever that player's actual loss were at the time.

We now know that for one hand it is appropriate to rebate 1.186% of actual player loss, whereas at 62,500 hands, 50% of actual loss may be repaid with both scenarios maintaining a 50% equivalency relative to theoretical loss in an even money game.

To determine points in between these extremes of number of hands, it is necessary to determine the conditional mean for each number of hands. To crudely demonstrate the process of integration the following is provided:-

Number of hands N = 100
Edge = 1.2%
Even money game

The mean = 1.2% x 100
= 1.2

1 standard deviation = square root (N)
= square root (100)
= 10

From basic statistics we know that 34.13% of results occur between the mean and one standard deviation.

13.64% of results occur between one and two standard deviations and

2.23% of results are greater than two standard deviations

From this we may roughly calculate the conditional mean for all player losses.

To do this we take the probability range and multiply this by the mid point result.

34.13% x {(1.2 + (1.2 + 10)) / 2 }
13.64% x {((1.2 + 10) + (1.2 + (2 x 10))) / 2 }
2.23% x {((1.2 + (2 x 10)) + (1.2 + (3 x 10))) / 2 }

and sum these which provides the conditional mean greater than the mean and then add the probability of results between 0 and the mean multiplied by that midpoint.

Without referring to normal distribution tables this may be approximated by taking the mean divided by the standard deviation and multiplying this by 34.13% and then multiplying that by the midpoint of zero and the mean.

= 1.2 / 10 x 34.13% x 1.2/2

Thus the conditional mean = 2.116
+ 2.210
+ 0.584
+ 0.025
= 4.935

This compares to the standard mean (theoretical loss) of 1.2 and thus if a 50% rebate on theoretical loss were desired the rebate on actual loss based upon the above would be

Rebate on actual loss % = 50% x 1.2 / 4.935
= 12.16%

As stated this is a very crude example provided for demonstration purposes only.

To more accurately calculate the percentage to be rebated it is merely necessary to utilise smaller sections when integrating and refer to normal distribution tables for the probabilities or to utilise a lesser known statistical function referred to as the "UNLLI" or Unit Normal Linear Loss Integral. This is basically analogous to the sum of all possible values of a standard normal variables positive distances above the number "a", multiplied by their corresponding probabilities of occurrence.

To put this into practice the following steps may be followed:-

1. Find expected loss for the player.

2. Find standard deviation of player result.

3. Calculate z = expected loss / standard deviation.

4. Look up UNLLI table corresponding to z (see table below).

5. Multiply UNLLI value by standard deviation.

6. Add number calculated from point 5 to number calculated from point 1.

7. Take whatever percentage of point 1 is to be returned and divide by the result of point 6.

Example:-

Hands = 750

Edge = 1.25%

Payouts = even money

Theoretical loss equivalency = 50%

1. 750 x 1.25% = 9.375

2. square root (N) 750 = 27.386

3. 9.375 / 27.386 = 0.342

4. UNLLI = 0.2508

5. 27. 386 x 0.2508 = 6.868

6. 9.375 + 6.868 = 16.243

7. 9.375 x 50% / 16.243 = 28.859%

UNIT NORMAL LINEAR LOSS INTEGRAL

Z	.00	.02	.04	.06	.08
0.00	.3989	.3890	.3793	.3697	.3602
0.10	.3509	.3418	.3329	.3240	.3154
0.20	.3069	.2986	.2904	.2824	.2745
0.30	.2668	.2592	.2518	.2445	.2374
0.40	.2304	.2236	.2170	.2104	.2040
0.50	.1978	.1917	.1857	.1799	.1742
0.60	.1687	.1633	.1580	.1528	.1478
0.70	.1429	.1381	.1335	.1289	.1245
0.80	.1202	.1160	.1120	.1080	.1042

0.90	.1004	.0968	.0933	.0899	.0866
1.00	.0833	.0802	.0772	.0742	.0714
1.10	.0686	.0660	.0634	.0609	.0585
1.20	.0561	.0539	.0517	.0496	.0475
1.30	.0456	.0437	.0418	.0401	.0383
1.40	.0367	.0351	.0336	.0321	.0307
1.50	.0293	.0280	.0268	.0256	.0244
1.60	.0233	.0222	.0212	.0202	.0192
1.70	.0183	.0174	.0166	.0158	.0150
1.80	.0143	.0136	.0129	.0122	.0116
1.90	.0110	.0104	.0099	.0094	.0089
2.00	.0084	.0080	.0075	.0071	.0067
2.10	.0063	.0060	.0056	.0053	.0050
2.20	.0047	.0044	.0042	.0039	.0037
2.30	.0036	.0034	.0032	.0030	.0028
2.40	.0027	.0026	.0024	.0023	.0022
2.50	.0021	.0018	.0017	.0016	.0016

For the game of Baccarat it is then possible to calculate the following table:-

BACCARAT (PLAYER/BANK 50% EQUIVALENT)

HANDS	% REBATE ON ACTUAL LOSS
10	4.78%
50	9.96%
100	13.52%
200	17.96%

300	21.01%
400	23.47%
500	25.39%
600	27.06%
700	28.37%
800	29.54%
900	30.78%
1000	31.75%
1100	32.63%
1200	33.29%
1300	34.06%
1400	34.79%
1500	35.48%
1600	35.96%
1700	36.58%
1800	37.01%
1900	37.57%
2000	37.95%
2500	39.89%
3000	41.34%
3500	42.44%
4000	43.48%
4500	44.18%
5000	44.88%

The above table for Baccarat is interesting in that it depicts the percentages of loss which can be paid for various numbers of hands to maintain a 50% rebate on theoretical loss equivalence. From this it can be seen that quite attractive rebates may be paid. A central question which arises, however, is what is the best or simplest manner by which to calculate the number of hands. While a simple method may be to take time played and employ standard decision rates, that is inappropriate due to potentially widely divergent bet levels.

This is important because when dealing with actual loss some bets may be statistically insignificant. To demonstrate using extremes if we had 1000 hands with bets of $1000 and one hand with a bet of $1,000,000 clearly the player's final result will be primarily determined by whether they win or lose the $1,000,000 hand. It can be said therefore that the 1000 hands are insignificant. Thus a reasonable method of calculating the number of hands played for the purposes of determining a rebate on loss is to divide the total turnover by the player's maximum bet. This criteria may be particularly useful when the Casino permits a table differential to be employed which potentially allows an unlimited maximum bet to be placed.

Determining the maximum bet placed is generally a simple proposition if dealing with an individual player. In a junket group situation where members of the same group may for example bet against each other on Baccarat, the bet could be considered to be the difference between the opposing bets, even though the turnover is the sum of the opposing bets.

For other non-even pay off games such as roulette, the mathematics remains similar, however, because the player wins more when they do win but this occurs less often, the percentage of actual loss which may be rebated is relatively less.

To incorporate this factor into the previously shown formula requires the calculation of the variance for a specific game for one result. In an even money game such as Baccarat (when playing Bank or Player) the variance may be approximated as one and therefore the previously shown formula was valid.

In any game the calculation of variance is accomplished by summing the square of the player wins multiplied by the probability of the returns. The standard deviation then becomes the square root of the number of hands multiplied by the average squared result.

In a game with multiple betting options at the same game with varying payoffs but the same house advantage (eg Roulette) the variance figure utilised when calculating a rebate on loss would necessarily be the maximum figure.

The appropriate numbers for various games are:-

Baccarat = 1 (exact figures 1.00 player)
0.95 bank)

Blackjack = 1.26

Roulette = 34.1 (single number bets on single zero roulette)

In games with multiple betting options at varying payoffs and house edges it would be appropriate to fully calculate the rebate payable on every option and utilise the variance from the result which returns the least to the player as otherwise any requirements on data collection by staff may be prohibitive.

When performing this calculation the following formula may be used:-

1. Calculate the variance for one play.

2. Find expected loss for the player.

3. Find standard deviation of player result = square root of (hands multiplied by variance (refer point 1))

4. Calculate z = expected loss / standard deviation.

5. Look up UNLLI table corresponding to z.

6. Multiply UNLLI value by standard deviation.

7. Add number calculated from point 5 to number calculated from point 2.

8. Take whatever percentage of point 2 is to be returned and divide by the result of point 7.

This then produces the following example of a table of rebate percentages applicable to be paid for the game of roulette (when playing single numbers on a single zero game) and which maintains a 50% equivalence on theoretical loss.

ROULETTE (SINGLE NUMBER PLAY ON SINGLE ZERO ROULETTE)

HANDS	% REBATE ON ACTUAL LOSS
10	1.79%
50	3.93%
100	5.44%
200	7.52%
300	9.10%
400	10.33%
500	11.39%
600	12.33%
700	13.19%
800	13.98%
900	14.58%
1000	15.27%
1100	15.93%
1200	16.55%

1300	17.00%
1400	17.57%
1500	17.97%
1600	18.50%
1700	19.01%
1800	19.35%
1900	19.83%
2000	20.14%
2500	22.00%
3000	23.48%
3500	24.79%
4000	25.98%
4500	27.07%
5000	27.91%

What use is all this information?

Many would argue that this is all too complicated to be of practical application in the Casino industry.

Firstly, it provides a mechanism by which any existing rebate on loss policy can be analysed to assess the long term or theoretical cost to the business.

Secondly, in the high level junket segment it provides a means by which variable percentage rebates on loss can be structured which can be both attractive and may be combined with rebates on turnover or the provision of other complimentaries. Being criteria based any policies so developed would possess a long term validity.

Thirdly, it provides a challenge to incorporate a rebate on loss element into the standard calculation of premium player complimentaries. One of the basic limitations of a theoretical loss based complimentary system is that while fine in theory the players often complain that no consideration is given should they incur a substantial loss. To any player, funds are a limiting factor which if depleted will limit the turnover they can provide which may also mean that what would normally be comped, to add insult to injury they may have to pay for. Some complimentary policies address this in a superficial way but again these are not criteria based.

To say that incorporating the above formula into a player rating system would not be practical because it could not be calculated or would not be understood by the player is incorrect. Most player rating systems in large Casinos are computerised which would certainly enable any calculation to be undertaken.

Secondly players already take most things on trust in terms of what complimentaries are provided. For example the decision rates per hour, house edges, average bet levels and percentage of theoretical loss returned are generally unknowns from the player's perspective. Therefore if the objective is to find the most equitable system upon which to base complimentaries some aspect of player loss should be incorporated, and from a business perspective that should equate to a standard theoretical cost.

Structuring a program of this nature could be achieved by adding a rebate on theoretical loss to a percentage rebate on actual loss, providing either depending upon which is the greater of the two or relying solely on one or the other. Of course as in any player rating system success relies heavily on capturing good data initially. To do this it is imperative that the gaming staff performing this function realise its importance.

Finally if referring to UNLLI tables etc is still considered too complex then the following approximation of a rebate on loss percent calculation may be of use:-

b = a. Y square root (V.N) x 100

0.5Y square root (V.N) + 0.17 Y2 + 0.4 V.N

where a = the percentage of theoretical loss equivalent

where b = the percentage of actual loss

where Y = theoretical loss to the player

where N = the number of hands played (turnover/maximum bet)

where V = the average squared result for one game

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